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Enhanced coverage through relay assisted carrier aggregation for cellular networks

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ABSTRACT

Carrier aggregation (CA) is an important feature of next generation cellular networks to meet the growing demand for wireless broadband services from fast-growing mobile users. This also facilitates the efficient use of fragmented spectrum among users of varying data rates. However, this does bring in a number of new implementation challenges, among which energy efficiency along with ensuring coverage and capacity fairness for users becomes the fundamental challenge for such systems. This paper initially analyses the outage capacity of such systems for both intra- and inter-band CA. Then, relaying is proposed as an energy efficient technique for enhancing the capacity and coverage of such systems. In this context, the ergodic rate of a typical user is analysed for both single- and multi-flow relay assisted CA systems. Different band deployment configurations are also analysed and compared. Further, it is observed that relays can be utilised to boost the capacity of lower frequency component carriers ensuring better capacity fairness among the users.

1. Introduction

The unprecedented demand for wireless broadband services has led to a profound paradigm shift in cellular networks to keep pace with the user demand. In order to achieve these high data rates, single carrier capacity is no longer adequate. In addition, the limitation of available contiguous spectrum being allocated and licensed makes multi-carrier usage an attractive proposition. In this context, carrier aggregation (CA) where multiple component carriers are aggregated to expand the effective bandwidth is considered as a key feature of next generation cellular systems [1–6]. This can effectively address the requirement of large transmission bandwidths (20–100 MHz) and high peak data rates (up to 500 Mbps in the uplink and 1 Gbps in the downlink).

The bandwidth of the individual component carriers constituting carrier aggregation can vary widely (ranging from 1.4 MHz to 20 MHz for LTE carriers) and can be contiguous or non-contiguous in frequency within the same band (intra-band CA) or across multiple bands (inter-band CA). This enables access to a very large bandwidth as well as facilitates the efficient use of fragmented spectrum [4,7–10]. However, the radio channel characteristics and transmission performance, such as propagation path loss and Doppler shift of component carriers vary significantly based on the operating frequency [11]. This would require appropriate resource allocation and load balancing across the carriers to achieve better resource utilization and spectral efficiency. It further reduces the probability of having unused resources, thereby improving the network efficiency and user performance by dynamically allocating traffic across the entire spectrum. However, such aggregation of multiple spectral bands conspires to present growing challenges to the equipment vendors and device manufacturers developing multi-frequency (and increasingly multimode products).

In practice, the spectrum allocation for CA is governed by complexity, cost, capability, and power consumption. There will be additional complexity in radio frequency (RF) implementation based on the number of RF chains employed for CA [7]. This makes intra-band CA with contiguous spectrum the least complex architecture, since it has only a single transmitter chain, and inter-band CA the most complex (each carrier has its own transmitter chain). However, practical deployments often have spectrum dispersed across several bands. Therefore, the non-contiguous CA technique enables mobile network operators to fully utilize their current spectral resources, including the unused scattered frequency bands and those already allocated for some legacy systems. However, the energy consumption as well as the signal processing complexity increases with the number of RF chains [12]. Studies have also shown that transmission over multiple carriers will result in an increase of

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peak-to-average power ratio (PAPR) and cubic metric (CM) [13]. For energy constrained users, the cost of transmitting at (or close to) maximum power will counterbalance the gain brought by CA. This introduces some new challenging issues related to radio resource management in CA based systems.

The feasibility of employing dual carriers for high speed packet access (HSPA) networks in the uplink has been studied in [14,15]. It is shown that, power limited users at the cell edge can barely benefit from their dual carrier capabilities [14] and the gain from CA is lower in the uplink than in the downlink [15]. The additional power requirement for CA is particularly intensified for UEs at the cell edge due to severe propagation conditions which result in less number of component carriers with good channel quality [16]. Further, aggregation of diverse carriers with large frequency separation will result in asymmetric coverage [2,3]. Generally, a lower-frequency carrier can provide larger service coverage, hence suitable for supporting higher order modulation and coding schemes. Therefore, improving the energy efficiency along with ensuring coverage and capacity fairness for users becomes the fundamental challenge for CA based systems.

Recently, relaying has been studied as a promising feature to reduce energy consumption [17,18], extend coverage [19,20], increase capacity and provide cost effective deployment options for future cellular networks [21,22]. Relays ($\mathcal{R}$) are low power nodes that have much smaller coverage compared to base stations (BS). It provides much higher average signal-to-interference-plus-noise ratios (SINRs) which is especially beneficial to meet the quality of service (QoS) requirements of cell-edge users. Consequently, relays provide an energy efficient and economical means that are easy to be deployed without modifying current cellular infrastructure. Fixed relays for capacity and coverage enhancement has already been standardised in IEEE802.16j. In this paper, the focus is on analysing relaying as an energy efficient technique for enhancing the capacity and coverage of CA based systems.

This paper considers a stochastic geometry framework for analysing the performance of relay assisted CA based systems. Initially, the probability of achieving the spectral efficiency by typical user equipment (UE) is measured by analysing the outage capacity. This basically quantifies the coverage area within which the desired spectral efficiency can be delivered by BSs and serves as the intermediary for designing advanced techniques to enhance this coverage. In this paper, relays are incorporated into the network for enhancing the coverage of CA based systems. The topic of relays for enhancing the coverage and capacity of cellular networks has been well studied in the past [23–27]. However, this paper analyses this aspect for CA based systems by analysing the rate which is a paramount metric for such systems. The rate is quantified from the aggregated carriers where the UE can be associated with either BS/relays or both across these carriers. This leads to *single flow* scheme where the UE is associated with either of BS or relays and *multi-flow* scheme by associating with both BS and relays across the carriers. The advantage of utilising multi-flow scheme with inter-band CA is that the operator can utilise the frequency spectrum at different bands based on their propagation characteristics. This favours utilising multi-flow association scheme with relays to boost the bands with lower coverage thereby mitigating the asymmetric coverage scenario arising in such systems. This paper analyses different band deployment configurations and the results will be informative for system design. The main contributions from this work can be summarized as follows:

1) Outage capacity analysis of CA systems – In Section 3, the outage capacity analysis of CA based systems is introduced for both intra- and inter-band CA. This basically delineates the coverage area for CA based systems to deliver high data-rates. It is observed that typically the cell-edge users are in outage due to the lack of carriers with good channel quality.

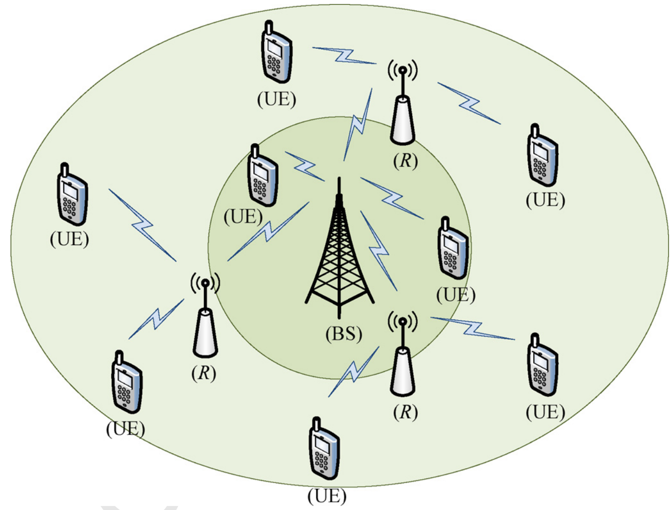

Fig. 1. Relay ($\mathcal{R}$) assisted cellular network model for energy efficient CA.

2) Relaying for greater coverage and capacity fairness in CA systems – Relaying is proposed as a technique to boost the capacity and eliminate the weak coverage of certain carriers for CA. It is observed that this provides greater coverage and capacity fairness among users. An analysis on the average ergodic rate of relay assisted CA systems is presented in Section 4. The analysis is extended to both single-flow and multi-flow CA in such systems.

3) Design Insights – From the analytical results, several observations may be informative for system design. In this context, relaying is found to be an energy efficient technique to enhance the coverage and capacity fairness in CA based systems. However, the ergodic rate saturates with a certain density of relay nodes and this rate saturation point is reached when almost all users are attached to at least one relay node.

The remainder of the paper is organized as follows. The framework for analysing the performance of relay incorporated CA based systems is provided in Section 2. In Section 3, the outage capacity of CA based systems is analysed. This serves as the intermediary for Section 4 where the relay assisted CA based systems is proposed. Numerical results demonstrating the performance of such systems is presented in Section 5. Finally, concluding remarks and future extensions of the present work in Section 6 wrap up this paper.

2. Carrier aggregation framework

In this paper, a cellular network model consisting of base stations (BS), relays ($\mathcal{R}$) and user equipments (UE) as shown in Fig. 1 is considered. The focus of this paper is on the downlink scenario and other key aspects of the model are explained in the subsequent sections.

2.1. Network & channel model

The system considers a multi-cell-multi-user (MCMU) radio access network (RAN), where the BSs are deployed according to some homogeneous Poisson point process (PPP) ϕ_{BS} of intensity λ_{BS}, and $\mathcal{R}$s are located according to another independent homogeneous PPP ϕ_R of intensity λ_R. This basically represents the average number of BSs and $\mathcal{R}$s per unit area. The UEs, which are randomly distributed based on an independent PPP of intensity λ_U, exploits the SINR advantage through relaying. Decode-and-forward (DF) based relaying protocol is considered in this paper due to its low channel state information (CSI) estimation complexity and relatively strong system level performance [28]. In this paper, relays

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use the same spectrum and protocols as the base station, which requires additional mechanisms to avoid the cross-tier interference. However, this has the advantage of letting UE connect to the relays as if it were a traditional BS. One method to mitigate the resulting cross-tier interference is by proper scheduling and allocating much lower power for $\mathcal{R}$s compared to BSs. Proper scheduling is achieved by operating the relays in transparent mode, whereby the BS is responsible for scheduling the packet transmission of each user with knowledge of the relay deployment. This is achieved by keeping track of potential relay nodes through the neighbouring cell list (NCL) [29]. When BS has data to be transmitted, it will initiate the transmission process by sending a request to the nearest $\mathcal{R}$, the relay that can best assist in delivering its message to the UE, from the NCL. However, BSs will be assigned the task of delivering the message to UE during instances when it is the nearest site or on absence of relay nodes.

Transmission in carrier aggregation based systems is performed by employing the set of B available spectral bands denoted as $\zeta = 1, 2, \cdots, B$, with each band i having a bandwidth and path loss exponent denoted as b_i and α_i, respectively. Each spectral band is assumed to have relatively constant path loss exponent across it. However, different path loss exponents are assumed for different bands to capture the possibly large differences in propagation characteristics associated with each band's carrier frequency [11]. As a result, the performance of the network depends on the different frequency bands employed for CA. Considering an efficient band deployment configuration, the SINR at UE j for transmissions from site k in band i with interference from the other sites transmitting in the same band i (except the site k) can be computed as

$$SINR_i^{kj} = \frac{P_i^k H_i^{kj} C_i ||\delta^{kj}||^{-\alpha_i}}{\sum_{l\in K}\sum_{s\in\hat{\phi}_i^l\setminus k} P_i^l H_i^{sj} C_i ||\delta^{sj}||^{-\alpha_i} + N_i} \quad (1)$$

where $K = \{BSs, \mathcal{R}s\}$ denotes the transmitting site from $\phi_K = \{\phi_{BS}, \phi_R\}$ set of PPP distributed transmission sites, $\hat{\phi}_i^l$ is the PPP of density $\theta_i^K \lambda_K$ thinned from ϕ_K with θ_i^K referring to the load of K in band i (fraction of sites in K using band i) and λ_K is the corresponding density, P_i^k is the transmit power of site k in band i, H_i^{kj} is the channel fading coefficient from the kth site to the jth UE in band i, C_i is a constant that specifies the path loss in band i when the link length is 1, $||\delta^{kj}||$ denotes the Euclidean norm of δ^{kj}, the distance between kth site and the jth UE, and $N_i = b_i N_0$ with N_0 being the power spectral density of the background noise. Note that C_i strongly depends on the carrier frequency, e.g. $C_i \cong (w_i/4\pi)^2$ where w_i denotes the wavelength of the frequency band i [11]. The channel fading coefficients H_i^{kj} are assumed to be independently and identically Rayleigh distributed and shadowing is ignored for simplicity. However, the randomness of site locations considered in the proposed model actually emulates shadowing [30].

2.2. Spectrum aggregation scenarios

In spectrum aggregated systems, the performance is dependent on the attributes of the individual components carriers. Based on the selected component carriers, CA can be differentiated into

2.2.1. Intra-band CA

In this scenario, the individual component carriers for CA are selected from the same operating frequency band [3]. The component carriers can then be adjacent to each other (contiguous) or separated along the frequency band (non-contiguous). Even though former scenario is the simplest CA technique, the latter might be more realistic due to the fragmented spectrum available today.

2.2.2. Inter-band CA

The component carriers in this scenario belong to different operating frequency bands [3]. This type of aggregation can potentially exploit the different coverage footprints available due to the diverse radio propagation characteristics of different bands as well as fully utilize the spectral resources. But this form of CA will also require additional complexity due to multiple non-contiguous RF chains.

In this paper, the focus is on both intra- and inter-band CA for enhancing the downlink peak user data rates and spectral utilization. Similar analysis can be extended to gauge the performance in the uplink. However, there are practical difficulties in employing multiple carriers at the UE due to power limitations considering realistic device linearity.

2.3. Resource organization

The available carriers (intra- or inter-band) can be aggregated from either a single site (BS or $\mathcal{R}$) or across non co-located sites ($\mathcal{R}$ and BS). The former is referred to as single flow CA and the latter as multi-flow CA [11,31]. In single flow CA, the relays utilise all or a fraction of the available bands used by the BS to guarantee the required QoS at the UE. However, as both sites use the same carriers, interference management techniques need to be employed to avoid self-interference. In multi-flow CA, relays are employed to boost certain carriers only. This is similar to frequency selective repeaters that are deployed in current networks. Multi-flow might be more suited for relay assisted CA in reality, where UE is configured with a primary component carrier from the BS to support mobility and additional secondary component carriers are only added to enhance the data-rate as and when required. However, the propagation delays across direct and relayed carriers may be different, thus requiring appropriate transmission timing control. Such orthogonal deployment of carriers – different tiers use different bands is considered for multi-flow scenario to highlight the advantage of relays which can be utilised to boost certain carriers. The focus of this paper is on analysing relay assisted carrier aggregation and synchronization issues are considered beyond the scope of the present work.

The typical UE scans over all the bands supported by both BS and $\mathcal{R}$, and associates with either one or both sites based on the technique employed. However, considering each band to have relatively constant propagation characteristics, single- and multi-flow will coincide resulting in selecting the same site (BS or $\mathcal{R}$) for intra-band CA. Thereby, only inter-band CA with multi-flow association scheme will permit the UE to perform CA across all the available bands from the different sites. In this paper, the focus is on intra- and inter-band CA along with the corresponding association schemes.

3. Outage capacity of CA systems

Carrier aggregation allows scalable expansion of effective bandwidth in order to support the high data rate requirements, thereby making transmission rate to be a paramount metric for such systems. In this context, this section evaluates the outage capacity of such systems, which is defined as the probability that the transmission rate falls below a specified threshold i.e., the desired spectral efficiency R_{th} [32]. This can be expressed as

$$P(R \le R_{th}|B) = P\left(\sum_{i=1}^{B} R_i \le R_{th}\right) \quad (2)$$

where R refers to the total transmission rate and R_i the rate of individual band, i. The present analysis basically delineates the coverage area of CA based systems and serves as the intermediary

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for designing relaying based approach for enhancing the coverage of such systems. Hence, the present analysis is limited to single tier networks but can be easily extended to multi-tier networks. Considering individual component carriers with bandwidth b_i, the available throughput can be simplified using Shannon–Hartley theorem as

$$P\left(\sum_{i=1}^{B} R_i \le R_{th}\right) = P\left(\sum_{i=1}^{B} b_i \log_2(1+SINR_i) \le R_{th}\right) = 1 - P\left(\sum_{i=1}^{B} b_i \log_2(1+SINR_i) \ge R_{th}\right) \quad (3)$$

where SINR for band i has been evaluated in (1). Equation (3) is analysed for intra- and inter-band CA as given below.

3.1. Intra-band CA

Considering uniform propagation characteristics across a band of frequencies, the SINRs for intra-band CA can be assumed to be equal. Then, (3) can be simplified as

$$P\left(\sum_{i=1}^{B} b_i \log_2(1+SINR_i) \ge R_{th}\right) = P\left(W\log_2(1+SINR) \ge R_{th}\right) = P\left(SINR \ge \left(2^{R_{th}/W} - 1\right)\right) \quad (4)$$

where $W = \sum_{i=1}^{B} b_i$. This can be evaluated by considering interference power to be exponentially distributed as [33]

$$P(SINR \ge T) = \pi\lambda_{BS} \int_0^\infty \exp\left(-\pi\lambda_{BS} x(1+\beta(T,\alpha)) - \frac{T x^{\alpha/2}}{\sigma}\right) dx \quad (5)$$

where $T = \left(2^{R_{th}/W} - 1\right)$, $\sigma = \frac{C_i P_i^{BS}}{W N_0}$, and $\beta = T^{2/\alpha} \int_{T^{-2/\alpha}}^{\infty} \frac{1}{1+u^{\alpha/2}} du$.

3.2. Inter-band CA

In inter-band CA, non-adjacent component carriers belonging to different bands are aggregated which allows the exploitation of a fragmented spectrum. However, this potentially leads to differing frequency characteristics of individual bands. In this context, equation (3) can be evaluated as

$$P\left(\sum_{i=1}^{B} b_i \log_2(1+SINR_i) \ge R_{th}\right) \le \frac{\mathrm{E}\left[\sum_{i=1}^{B} b_i \log_2(1+SINR_i)\right]}{R_{th}} \quad (6)$$

where the simplification is based on Markov's inequality, $P(X \ge x) \le \mathrm{E}(X)/x$ [34]. In this case, the typical UE is associated with the nearest BS that provides the strongest received power for a particular band. The ergodic rate for such a system can be obtained by summing the rates over the available bands and can be evaluated as [11]

$$\overline{R} = \mathrm{E}\left[\sum_{i=1}^{B} b_i \log_2(1+SINR_i)\right] = \sum_{i=1}^{B} 2\pi\lambda_{BS} b_i \int_0^\infty \int_0^\infty \frac{1}{1+x} \times \exp\left(-\frac{x\delta^{\alpha_i}}{\sigma} - \pi\lambda_{BS}\left(\rho\left(x,\alpha_i\right)+1\right)\delta^2\right)\delta\, d\delta\, dx \quad (7)$$

where $\rho(x,\alpha_i) = \int_1^\infty \frac{x}{x+y^{\alpha_i/2}} dy$. By evaluating (6) using (7), the outage capacity of inter-band CA can be computed which basically serves as the lower bound for such systems.

From the analysis, it is evident that CA based systems offers boost to the data rates for only those users that can achieve the desired spectral efficiency. Usually, those UEs present at the cell-edges, termed edge UEs, are in outage due to the presence of less number of CCs with good channel quality. In this context, advanced techniques are required to boost the capacity and eliminate the weak coverage for such CA based systems especially for edge UEs. This paper analyses integrating relaying technique with such systems for enhancing the capacity and coverage.

4. Relay assisted CA

Relaying techniques have been extensively analysed for mesh and ad-hoc networks [35,36]. Recently, relay assisted cellular networks are being considered as an economical means of enhancing the capacity and coverage [37]. In such networks, UEs have the diversity benefit of two possible links, the direct link to the BS, and a link via relay. The introduction of relays also increases the coverage radius by providing higher SINR to the edge UEs. Thus, the infrastructure cost of deploying more BSs is reduced. Consequently, relaying technique can complement the increased power consumption necessitated through CA. In this context, this section analyses relay assisted CA based cellular networks for enhancing the capacity and coverage.

Another feature of relay assisted networks is that the exchange of information is via the BS, resulting in $\mathcal{R}$ to act as a BS in the perspective of UE and, from BS's view $\mathcal{R}$ will be seen as a UE. Furthermore, the interference arising due to relaying is limited to those relays that cooperate in the transmission. This interference becomes a harmful factor to the overall system throughput, especially as the number of relays increases and will require advanced interference cancellation techniques [38,39]. The average ergodic rate of UEs assisted by relay nodes is analysed in this section.

Average ergodic rate

The average ergodic rate for both single flow and multi-flow CA of relay assisted cellular networks is given as [40]

$$\overline{R} = \sum_{i=1}^{B} \sum_{k\in\{BS,R\}} R_i^k A_i^k \quad (8)$$

where A_i^k is association probability that a typical user is associated to the kth station (BS or $\mathcal{R}$) in band i and R_i^k is the corresponding ergodic rate of that user. For the present analysis, the UE is considered to be associated with the station that offers the strongest average power.

4.1. Relay assisted cellular networks – single flow CA

In single flow CA, a typical UE scans over all the bands from BS and $\mathcal{R}$, and connects to the station that provides the strongest average power in some band. Then, the probability that a typical user is associated with BS can be computed as (considering UE is associated with a single station for all bands, the band reference subscript can be removed for single-flow association probability analysis) [40]

$$A^{BS} = \mathrm{E}_{\delta^{BS}}\left[P\left[P_j^{BS} C_j ||\delta^{BS}||^{-\alpha_j} > P_i^{R} C_i ||\delta^{R}||^{-\alpha_i}, \forall(i,j) \in B\right]\right] \quad (9)$$

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Here, $(i, j) \in B$ is the bands employed by $\mathcal{R}$ and BS, respectively. The average received power in (9) is strongest when path loss exponent is the minimum. Hence, (9) is simplified by considering $i^* = \arg\min_{i\in B}\alpha_i$ and $j^* = \arg\min_{j\in B}\alpha_j$ as

$$A^{BS} = \mathrm{E}_{\delta^{BS}}\left[P\left[\delta^R > \left(\frac{P^R_{i^*}C_{i^*}}{P^{BS}_{j^*}C_{j^*}}\right)^{1/\alpha_{i^*}}\left(\delta^{BS}\right)^{\alpha_{j^*}/\alpha_{i^*}}\right]\right] = \int_0^\infty P\left[\delta^R > \left(\frac{P^R_{i^*}C_{i^*}}{P^{BS}_{j^*}C_{j^*}}\right)^{1/\alpha_{i^*}} r^{\alpha_{j^*}/\alpha_{i^*}}\right] f_{\delta^{BS}}(r)\,dr \quad (10)$$

The probability $P\left[\delta^R > \left(\frac{P^R_{i^*}C_{i^*}}{P^{BS}_{j^*}C_{j^*}}\right)^{1/\alpha_{i^*}} r^{\alpha_{j^*}/\alpha_{i^*}}\right]$ and the probability density function (PDF) of δ^{BS} ($f_{\delta^{BS}}(r)$) can be derived using the simple fact that the null probability of a 2-D Poisson process with density λ in an area A is $\exp(-\lambda A)$ [33]. Then,

$$P\left[\delta^R > \left(\frac{P^R_{i^*}C_{i^*}}{P^{BS}_{j^*}C_{j^*}}\right)^{1/\alpha_{i^*}} r^{\alpha_{j^*}/\alpha_{i^*}}\right] = \exp\left(-\pi\lambda_R\left(\frac{P^R_{i^*}C_{i^*}}{P^{BS}_{j^*}C_{j^*}}\right)^{2/\alpha_{i^*}} r^{2\alpha_{j^*}/\alpha_{i^*}}\right) \quad (11)$$

and

$$f_{\delta^{BS}}(r) = 2\pi\lambda_{BS} r \exp\left(-\pi\lambda_{BS}r^2\right) \quad (12)$$

Combining (9)–(12) yields

$$A^{BS} = 2\pi\lambda_{BS}\int_0^\infty \exp\left(-\pi\left(\lambda_R\left(\frac{P^R_{i^*}C_{i^*}}{P^{BS}_{j^*}C_{j^*}}\right)^{2/\alpha_{i^*}} r^{2\alpha_{j^*}/\alpha_{i^*}} + \lambda_{BS}r^2\right)\right) r\,dr \quad (13)$$

Similarly, the association probability for a typical UE with $\mathcal{R}$ can be computed as

$$A^{R} = 2\pi\lambda_{R}\int_0^\infty \exp\left(-\pi\left(\lambda_{BS}\left(\frac{P^{BS}_{j^*}C_{j^*}}{P^{R}_{i^*}C_{i^*}}\right)^{2/\alpha_{j^*}} r^{2\alpha_{i^*}/\alpha_{j^*}} + \lambda_{R}r^2\right)\right) r\,dr \quad (14)$$

Next, the average ergodic rate of a typical user associated with station k is computed by averaging the link rate over the distance and the channel fade between station k and the UE. This is defined as

$$R^k_i = \mathrm{E}_d\left[\mathrm{E}_H\left[b_i\log_2(1+SINR_i(\delta))\right]\right] = \int_0^\infty \mathrm{E}_H\left[b_i\log_2(1+SINR_i(\delta))\right] f_{\Delta_k}(\delta)\,d\delta \quad (15)$$

where $f_{\Delta_k}(\delta)$ is the PDF of the distance Δ_k between the user and the kth station.

Lemma 1. *The PDF $f_{\Delta_{BS}}(\delta)$ of the distance Δ_{BS} between a typical user and BS is*

$$f_{\Delta_{BS}}(\delta) = \frac{2\pi\lambda_{BS}}{A^{BS}}\delta\exp\left(-\pi\left(\lambda_R\left(\frac{P^R_{i^*}C_{i^*}}{P^{BS}_{j^*}C_{j^*}}\right)^{2/\alpha_{i^*}}\delta^{2\alpha_{j^*}/\alpha_{i^*}} + \lambda_{BS}\delta^2\right)\right) \quad (16)$$

and similarly for a typical user associated with $\mathcal{R}$ is

$$f_{\Delta_{\mathcal{R}}}(\delta) = \frac{2\pi\lambda_{R}}{A^{R}}\delta\exp\left(-\pi\left(\lambda_{BS}\left(\frac{P^{BS}_{j^*}C_{j^*}}{P^{R}_{i^*}C_{i^*}}\right)^{2/\alpha_{j^*}}\delta^{2\alpha_{i^*}/\alpha_{j^*}} + \lambda_{R}\delta^2\right)\right) \quad (17)$$

The proof is given in Appendix A.

Next the expectation over the fading fields is evaluated, by using the property $\mathrm{E}[X] = \int_0^\infty P[X > x]dx$ for $X > 0$ [40], as

$$\mathrm{E}_H[b_i\log_2(1+SINR_i(\delta))] = \int_0^\infty P[b_i\log_2(1+SINR_i(\delta)) > t]dt = \int_0^\infty P[SINR_i(\delta) > \psi(t)]dt \quad (18)$$

where $\psi(t) = (2^{t/b_i} - 1)$. Equation (18) can be evaluated by using (1) as

$$P[SINR_i(\delta) > \psi(t)] = P\left[\frac{P^k_i H^k_i \delta^{-\alpha_i}}{I_{\tilde{\phi}_{BS}} + I_{\tilde{\phi}_R} + N_i/C_i} > \psi(t)\right] = P\left[H^k_i > \frac{\psi(t)\delta^{\alpha_i}}{P^k_i}\left(I_{\tilde{\phi}_{BS}} + I_{\tilde{\phi}_R} + \frac{N_i}{C_i}\right)\right] \quad (19)$$

where $I_{\tilde{\phi}_{BS}} = \sum_{j\in\phi_{BS}\setminus k} P^{BS}_i H^j_i ||\delta^j||^{-\alpha_i}$ and $I_{\tilde{\phi}_R} = \sum_{s\in\overline{\phi}_R\setminus k} P^R_i H^s_i \times ||\delta^s||^{-\alpha_i}$. Here, $\overline{\phi}_R$ is the PPP of density $\tau_R\lambda_R$ thinned from ϕ_R, where τ_R is the probability that a typical relay is within the coverage of the BS, i.e. $\tau_R = P[SINR^R_i(x) \geq \gamma_{th}]$. This basically ensures that the interference in the network is from only those relays which have successfully decoded the information and thereby will be cooperating in the transmissions. τ_R can be evaluated using (5) with $T = \gamma_{th}$.

Considering the channel to be Rayleigh distributed, the random variable H^k_i follows an exponential distribution with mean 1 denoted as $H^k_i \sim \exp(1)$ and thus $P(H^k_i > x) = \exp(-x)$. Therefore, equation (19) can be simplified as [40]

$$P\left[H^k_i > \frac{\psi(t)\delta^{\alpha_i}}{P^k_i}\left(I_{\tilde{\phi}_{BS}} + I_{\tilde{\phi}_R} + \frac{N_i}{C_i}\right)\right] = \exp\left(-\frac{\psi(t)\delta^{\alpha_i}N_i}{P^k_i C_i}\right) L_{I_{\tilde{\phi}_{BS}}}\left(\frac{\psi(t)\delta^{\alpha_i}}{P^k_i}\right) L_{I_{\tilde{\phi}_R}}\left(\frac{\psi(t)\delta^{\alpha_i}}{P^k_i}\right) \quad (20)$$

where $L_{I_{\tilde{\phi}_{BS}}}$ and $L_{I_{\tilde{\phi}_R}}$ are the Laplace transform of $I_{\tilde{\phi}_{BS}}$ and $I_{\tilde{\phi}_R}$, respectively. From [33,40]; this can be evaluated as

$$L_{I_{\tilde{\phi}_{BS}}}\left(\frac{\psi(t)\delta^{\alpha_i}}{P^k_i}\right) = \exp\left(-\pi\lambda_{BS}\left(\frac{P^{BS}_i}{P^k_i}\right)^{2/\alpha_i}\rho(\psi(t),\alpha_i)\delta^2\right) \quad (21)$$

and

$$L_{I_{\tilde{\phi}_R}}\left(\frac{\psi(t)\delta^{\alpha_i}}{P_i^k}\right) = \exp\left(-\pi\tau_R\lambda_R\left(\frac{P_i^R}{P_i^k}\right)^{2/\alpha_i}\rho(\psi(t),\alpha_i)\delta^2\right) \tag{22}$$

where $\rho(\psi(t),\alpha_i) = \psi(t)^{2/\alpha_i}\int_{\psi(t)^{-2/\alpha_i}}^{\infty}\frac{1}{1+u^{\alpha_i/2}}du$. From (15), (16), (18), (20)–(22), the average ergodic rate for a typical user associated with BS can be given as

$$R_i^{BS} = \frac{2\pi\lambda_{BS}}{A^{BS}}\int_0^\infty\int_0^\infty \exp\Bigg(-\frac{\psi(t)\delta^{\alpha_i}N_i}{P_i^{BS}C_i} - \pi\Bigg(\lambda_{BS}\delta^2(1+\rho(\psi(t),\alpha_i)) + \lambda_R\Bigg(\tau_R\left(\frac{P_i^R}{P_i^{BS}}\right)^{2/\alpha_i}\rho(\psi(t),\alpha_i)\delta^2 + \left(\frac{P_{i^*}^R C_{i^*}}{P_{j^*}^{BS}C_{j^*}}\right)^{2/\alpha_{i^*}}\delta^{2\alpha_{j^*}/\alpha_{i^*}}\Bigg)\Bigg)\Bigg)\delta\, d\delta\, dt \tag{23}$$

This can be further simplified for intra-band CA as

$$R^{BS} = \frac{2\pi\lambda_{BS}}{A^{BS}}\int_0^\infty\int_0^\infty \exp\Bigg(-\frac{\psi(t)\delta^{\alpha_i}N_i}{P^{BS}C_i} - \pi\delta^2\Bigg(\lambda_{BS}(1+\rho(\psi(t),\alpha_i)) + \lambda_R\left(\frac{P_i^R}{P_i^{BS}}\right)^{2/\alpha_i}(1+\tau_R\rho(\psi(t),\alpha_i))\Bigg)\Bigg)\delta\, d\delta\, dt \tag{24}$$

Similarly, the average ergodic rate for a typical user associated with $\mathcal{R}$ can be computed by using (15), (17), (18), (20)–(22) as

$$R_i^R = \frac{2\pi\lambda_R}{A^R}\int_0^\infty\int_0^\infty \exp\Bigg(-\frac{\psi(t)\delta^{\alpha_i}N_i}{P_i^R C_i} - \pi\Bigg(\lambda_R\delta^2(1+\tau_R\rho(\psi(t),\alpha_i)) + \lambda_{BS}\Bigg(\left(\frac{P_i^{BS}}{P_i^R}\right)^{2/\alpha_i}\rho(\psi(t),\alpha_i)\delta^2 + \left(\frac{P_{j^*}^{BS}C_j}{P_i^R C_i}\right)^{2/\alpha_{j^*}}\delta^{2\alpha_{i^*}/\alpha_{j^*}}\Bigg)\Bigg)\Bigg)\delta\, d\delta\, dt \tag{25}$$

For intra-band CA this simplifies to

$$R^R = \frac{2\pi\lambda_R}{A^R}\int_0^\infty\int_0^\infty \exp\Bigg(-\frac{\psi(t)\delta^{\alpha_i}N_i}{P_i^R C_i} - \pi\delta^2\Bigg(\lambda_R(1+\tau_R\rho(\psi(t),\alpha_i)) + \lambda_{BS}\left(\frac{P_i^{BS}}{P_i^R}\right)^{2/\alpha_i}(1+\rho(\psi(t),\alpha_i))\Bigg)\Bigg)\delta\, d\delta\, dt \tag{26}$$

The average ergodic rate for a typical user can be computed by substituting (13), (23) and (14), (25); and (13), (24) and (14), (26) into (8) for inter-band and intra-band CA, respectively. Though these expressions are not closed-form, it can be efficiently computed numerically.

4.2. Relay assisted cellular networks – multi-flow CA

In multi-flow CA, the UE can be associated with both $\mathcal{R}$ and BS simultaneously by exploiting the different bands available for data aggregation. The bands can be cochannel deployed where all the bands are used by both BS and relays or orthogonally deployed where different bands are utilised by relays and BS. The orthogonal deployment will result in reduced interference by mitigating inter-tier interference but results in a capacity loss. However, this performance loss can be reduced by optimizing the coverage distribution whereby the relays can be utilised to boost the bands with lower coverage. The average ergodic rate analysis for cochannel and orthogonal deployment is given in the following section.

(i) Cochannel deployment

In cochannel deployment, each band is utilised by both relays and BSs. Thereby, each UE is associated with the station that provides the maximum received power for that particular band. Then, the probability that a typical user is associated with BS for band i can be computed from (13) by substituting $i^* = j^* = i$ as [40]

$$A_i^{BS} = 2\pi\lambda_{BS}\int_0^\infty \exp\left(-\pi\left(\lambda_R\left(\frac{P_i^R}{P_i^{BS}}\right)^{2/\alpha_i} + \lambda_{BS}\right)r^2\right)r\,dr \tag{27}$$

This can be further simplified by utilising a change of variable $r^2 = t$ to obtain

$$A_i^{BS} = \frac{1}{1+\left(\frac{\lambda_R}{\lambda_{BS}}\right)\left(\frac{P_i^R}{P_i^{BS}}\right)^{2/\alpha_i}} \tag{28}$$

Similarly, the association probability of a typical user associated with $\mathcal{R}$ for band i can be computed as

$$A_i^R = \frac{1}{1+\left(\frac{\lambda_{BS}}{\lambda_R}\right)\left(\frac{P_i^{BS}}{P_i^R}\right)^{2/\alpha_i}} \tag{29}$$

The average ergodic rate of a typical user associated with kth station for multi-flow CA with cochannel deployment can be computed similarly to the single-flow CA. However, as each band is associated with different stations, the PDF of the distance D_k between the user and the kth station need to be evaluated.

Lemma 2. *The PDF $f_{\Delta_{BS}}(\delta)$ of the distance Δ_{BS} between a typical user and BS is*

$$f_{\Delta_{BS}}(\delta) = \frac{2\pi\lambda_{BS}}{A_i^{BS}}\delta\exp\left(-\pi\left(\lambda_R\left(\frac{P_i^R}{P_i^{BS}}\right)^{2/\alpha_i} + \lambda_{BS}\right)\delta^2\right) \tag{30}$$

and similarly for a typical user associated with $\mathcal{R}$ is

$$f_{\Delta_{\mathcal{R}}}(\delta) = \frac{2\pi\lambda_R}{A_i^R}\delta\exp\left(-\pi\left(\lambda_{BS}\left(\frac{P_i^{BS}}{P_i^R}\right)^{2/\alpha_i} + \lambda_R\right)\delta^2\right) \tag{31}$$

The proof is given in Appendix B.

Therefore, the average ergodic rate of a typical user associated with BS in band i for multi-flow CA with cochannel deployment can be computed from (15), (30), (18), (20), (21) as

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JID:YDSPR AID:2084 /FLA [m5G; v1.204; Prn:22/02/2017; 11:53] P.7 (1-11)

$$R_i^{BS} = \frac{2\pi\lambda_{BS}}{A_i^{BS}} \int_0^\infty \int_0^\infty \exp\Bigg(- \frac{\psi(t)\delta^{\alpha_i} N_i}{P_i^{BS} C_i} - \pi\delta^2 \Bigg(\lambda_{BS}(1+\rho(\psi(t),\alpha_i)) + \lambda_R \Big(\frac{P_i^R}{P_i^{BS}} \Big)^{2/\alpha_i} (1+\tau_R \rho(\psi(t),\alpha_i)) \Bigg)\Bigg) \delta \, d\delta \, dt \quad (32)$$

Similarly, the average ergodic rate of a typical user associated with $\mathcal{R}$ in band i for multi-flow CA with cochannel deployment can be computed from (15), (31), (18), (20), (22) as

$$R_i^{R} = \frac{2\pi\lambda_{R}}{A_i^{R}} \int_0^\infty \int_0^\infty \exp\Bigg(- \frac{\psi(t)\delta^{\alpha_i} N_i}{P_i^{R} C_i} - \pi\delta^2 \Bigg(\lambda_{BS} \Big(\frac{P_i^{BS}}{P_i^{R}} \Big)^{2/\alpha_i} (1+\rho(\psi(t),\alpha_i)) + \lambda_R (1+\tau_R \rho(\psi(t),\alpha_i)) \Bigg)\Bigg) \delta \, d\delta \, dt \quad (33)$$

The average ergodic rate for multi-flow CA with cochannel deployment can be computed by substituting (28), (32) and (29), (33) into (8). These expressions can be easily evaluated numerically.

(ii) Orthogonal deployment

In orthogonal deployment, the relays and BSs utilise exclusive bands. The UE thereby aggregates over different bands based on the band associativity. As BSs and relays utilise different bands, the interference for a particular band is from the associated site i.e., there is no inter-site interference. Basically, BSs will cause interference to other BSs and relays will cause interference to other relays in a particular band. Similar to previous analysis, the probability that a typical user is associated with BS for band i is given as

$$A_i^{BS} = \begin{cases} 1 & \text{if band } i \text{ associated with BS} \\ 0 & \text{otherwise} \end{cases} \quad (34)$$

Similarly, the association probability of a typical user associated with $\mathcal{R}$ for band i can be given as

$$A_i^{R} = \begin{cases} 1 & \text{if band } i \text{ associated with } \mathcal{R} \\ 0 & \text{otherwise} \end{cases} \quad (35)$$

The average ergodic rate of a typical user associated with kth station for multi-flow CA with orthogonal deployment can be computed similarly as a single-tier network. Therefore, the average ergodic rate for a typical user associated with BS in band i can be computed as [11]

$$R_i^{BS} = 2\pi\lambda_{BS} b_i \int_0^\infty \int_0^\infty \frac{1}{1+x} \times \exp\Big(-\frac{x\delta^{\alpha_i}}{\sigma_{BS}} - \pi\lambda_{BS}(\rho(x,\alpha_i)+1)\delta^2 \Big) \delta \, d\delta \, dx \quad (36)$$

where $\sigma_{BS} = \frac{C_i P_i^{BS}}{W N_0}$ and $\rho(x,\alpha_i) = \int_1^\infty \frac{x}{x+y^{\alpha_i/2}} dy$ as specified in (7). Similarly, the average ergodic rate of a typical user associated with $\mathcal{R}$ in band i can be computed as

Table 1
Simulation parameters.

Density of macro BSs λ_{BS}	$(1000^2)^{-1}$ m^{-2}
Density of relays λ_R	$5\lambda_{BS}$
Density of UEs λ_U	$10\lambda_{BS}$
Max. Tx power of macro BS	46 dBm
Max. Tx power of relays	30 dBm
Noise PSD N_0	−172 dBm
SINR threshold for $\mathcal{R}$ to cooperate (γ_{th})	10 dB
Frequency bands (MHz)	[800, 1500, 2500]
Bandwidth b_i (MHz)	[1, 1, 1]
Path loss exponent α_i	[3, 3.5, 4]
Constant C_i	$[8.9, 2.5, 0.91] \times 10^{-4}$

$$R_i^{R} = 2\pi\lambda_{R} b_i \int_0^\infty \int_0^\infty \frac{1}{1+x} \times \exp\Big(-\frac{x\delta^{\alpha_i}}{\sigma_{R}} - \pi\lambda_{R}(\rho(x,\alpha_i)+1)\delta^2 \Big) \delta \, d\delta \, dx \quad (37)$$

where $\sigma_R = \frac{C_i P_i^R}{W N_0}$. The average ergodic rate for multi-flow CA with orthogonal deployment can be computed by substituting (34), (36) and (35), (37) into (8). These expressions can be easily evaluated numerically.

5. Numerical results

In this section, the analytical results developed for the outage probability and average ergodic rate of CA based systems are verified. Furthermore, a two-tier network consisting of BSs and $\mathcal{R}$s with 3 bands of frequencies as explained in Section 2 is extensively analysed. Both single- and multi-flow association schemes are analysed in this context with intra-band and inter-band CA. The simulation parameters are specified in Table 1.

5.1. Simulation environment

For simulation, the traditional hexagonal grid model and the random PPP model is considered. In hexagonal grid model, the home base station is considered to be located at the origin with 6 interfering base stations around the home base station. In PPP model, a square area of size 4 × 4 km^2 where $\lambda_{BS} = (1000^2)^{-1}$ m^{-2}, $\lambda_R = 5\lambda_{BS}$ and $\lambda_U = 10\lambda_{BS}$ is considered. With this deployment, there are on an average 16, 80, and 160 base stations (BS), relays ($\mathcal{R}$), and user equipments (UE) respectively within this area. Three bands of frequencies employed in LTE networks are utilised for the analysis and the varying propagation characteristics of each band is captured through different path loss exponents and constant C_i as explained in Section 2. Intra-band CA is performed on the 2500 MHz band as it supports much larger bandwidths compared to 800 and 1500 MHz bands which are utilised by 2G/3G services. For intra- and inter-band single-flow CA, cochannel band deployment is considered i.e., all bands are utilised by both relays and BSs. However, multi-flow inter-band CA scheme considers both cochannel and orthogonal band deployment.

5.2. Outage capacity

The outage capacity of intra-band and inter-band CA supported by macro BS is shown in Fig. 2. It is observed that outage capacity reduces with each additional band aggregated which confirms the higher data rate support through CA. For intra-band CA, it is observed that the results developed in Section 3 closely matches the simulation results for PPP model. However, the analysis performed in Section 3 is the lower bound for such a model with

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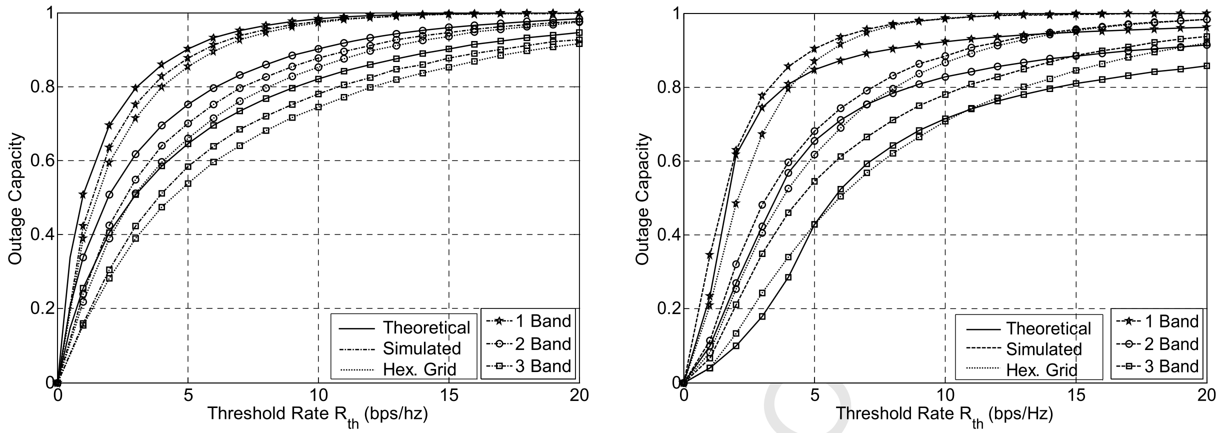
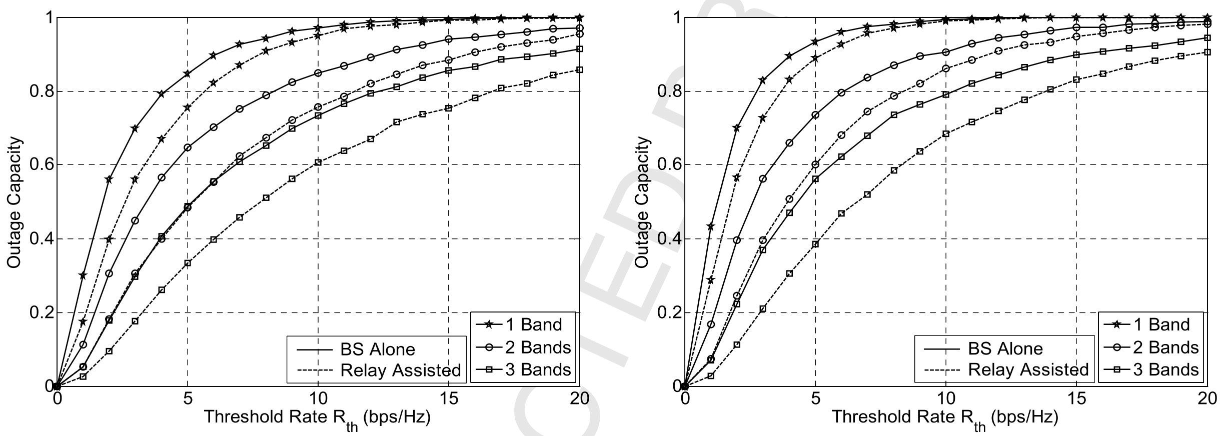


(a) Intra-band CA (b) Inter-band CA

Fig. 2. Outage capacity of intra- and inter-band CA systems.



(a) Intra-band CA (b) Inter-band CA

Fig. 3. Outage capacity of intra- and inter-band relay assisted single flow CA systems.

inter-band CA and the simulation results also concurs with this analysis. The outage capacity of hexagonal grid model is comparitively lower compared to the random Poisson model as a perfectly regular geometry will result in optimal performance [41].

Fig. 3 illustrates the performance improvement of outage capacity of intra-band and inter-band CA networks with relay assistance for single flow association scheme. The performance enhancement through relays increases with each additional band aggregated. Further, in Fig. 2 and 3 intra-band CA has reduced outage capacity compared to inter-band CA as intra-band CA utilises 2500 MHz with a higher path loss exponent compared to inter-band CA which utilises 800, 1500 and 2500 MHz bands. This can be attributed to the fact that dense networks with larger path loss exponents have better SINR coverage resulting in lower outage capacity [42].

The performance of single- and multi-flow association schemes employing inter-band CA is illustrated in Fig. 4. The band deployment configuration is given in closed braces with BS and $\mathcal{R}$ configuration separated by a semi-colon. The band deployment of 800 MHz, 1500 MHz and 2500 MHz can be represented as $[\mathrm{BS}_{800\ \mathrm{MHz}}, \mathrm{BS}_{1500\ \mathrm{MHz}}, \mathrm{BS}_{2500\ \mathrm{MHz}}; \mathcal{R}_{800\ \mathrm{MHz}}, \mathcal{R}_{1500\ \mathrm{MHz}}, \mathcal{R}_{2500\ \mathrm{MHz}}]$ and the allocation is represented by a 1 in the corresponding position. For example, $[1, 1, 1; 1, 1, 1]$ refers to universal cochannel band deployment between BS and relays; and $[0, 1, 1; 1, 1, 1]$ refers to 800 MHz band to be orthogonally allocated to the relays. From Fig. 4, it is observed that multi-flow scheme results in slight improvement in outage capacity for cochannel band deployment. However, multi-flow association scheme permits orthogonal band deployment which results in much lower outage capacity compared to single-flow association scheme by appropriate band allocation. From aggregating 800 and 1500 MHz bands it is observed that the outage capacity is minimized when 800 MHz band is orthogonally allocated to the relay. This further justifies the conclusion that is observed in Fig. 2 and 3 where a band with higher path loss exponent results in lowering the outage capacity. The performance of three bands CA where the

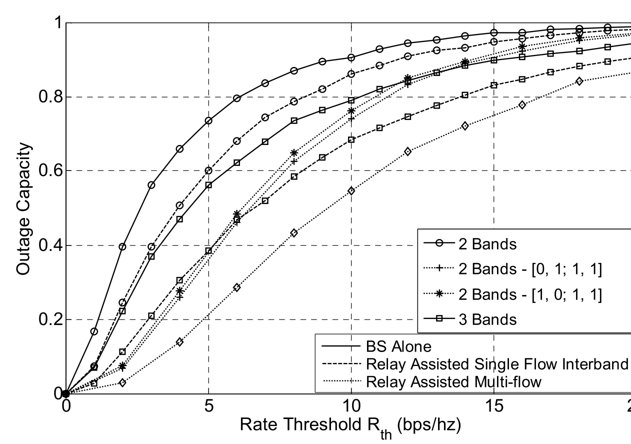


Fig. 4. Outage capacity comparison of inter-band single flow and multi-flow association schemes.

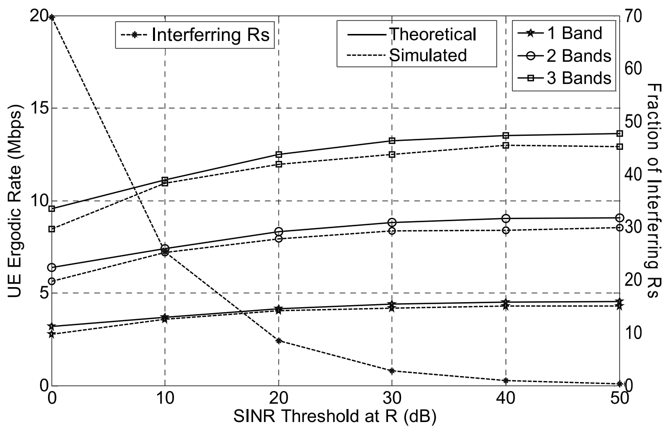


Fig. 5. Variation of average ergodic rate with SINR threshold at $\mathcal{R}$.

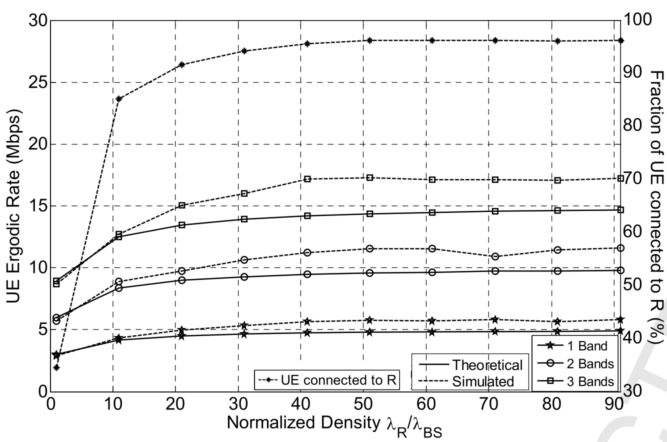


Fig. 6. Impact of density of relays on the UE ergodic rate.

lowest frequency band is orthogonally allocated to the relays is also shown in Fig. 4. From the results it can be observed that relays can be utilised to boost the performance of lower frequency component carriers which results in reduced outage capacity.

5.3. SINR threshold at $\mathcal{R}$

SINR threshold is utilised to determine the set of relay nodes that have accumulated sufficient mutual information to cooperate in the transmissions. Fig. 5 illustrates the variation of average UE ergodic rate with the SINR threshold. It is observed that higher values of SINR threshold results in lower number of cooperating relays which interferes with the transmission. This results in increased rate through the reduced interference. However, the rate saturates when the SINR threshold reaches a certain value as the fraction of interfering relays also decreases with the SINR threshold and the interference in the network arises from only the BSs. The theoretical results derived in Section 4 are also plotted in Fig. 5 and it is found to have a close match with the simulation results.

5.4. Normalized density of $\mathcal{R}$

Fig. 6 illustrates the variation of UE ergodic rate with normalized density of relay nodes by fixing the density of BSs and UEs. Initially, the ergodic rate increases almost linearly as the density of relays increases however it saturates for a certain density of relay nodes. The saturation point is reached when almost all the UEs are attached to the relay nodes as shown by the fraction of UEs connected to the relay nodes in Fig. 6. Even though the rate increases with additional bands aggregated, the rate saturation point almost

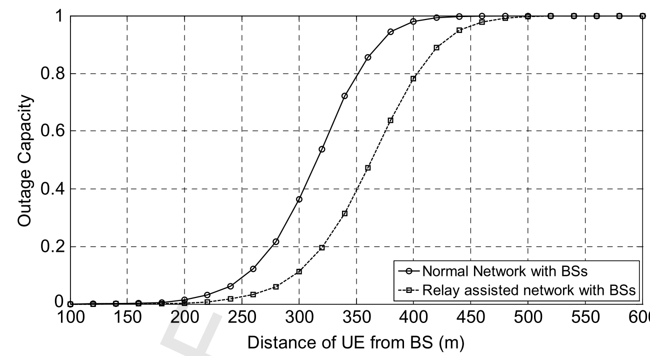


Fig. 7. Outage capacity with distance of UE from BS for traditional and relay assisted cellular networks.

remains constant irrespective of the number of bands aggregated. Fig. 6 also validates the theoretical results derived in Section 4 by observing a close match with the simulation results.

5.5. Performance with distance from BS

The hexagonal grid model is analysed to verify the coverage of relay assisted CA systems. The UE distance from the BS is varied and collinear placement of relay nodes is considered for simplicity of analysis. Fig. 7 illustrates the outage capacity where the threshold spectral efficiency is taken as 4 bps/Hz. It is observed that relay assisted CA provides better coverage. This further validates relaying as an energy efficient technique for enhancing the capacity and coverage of CA based systems.

6. Conclusions

This paper focuses on the energy efficiency of CA based systems. Initially, an analysis on the outage capacity of such systems is performed which basically delineates the coverage of such systems. This leads to exploiting relaying as an energy efficient technique to enhance the capacity and fairness of such systems. Outage capacity analysis is performed for such networks and the average ergodic rate of typical user associated with relay is also derived. Different band deployment configurations along with the association schemes are analysed and compared. The analysis has been verified through extensive simulation studies. The results reveal relay assisted CA to be an energy efficient technique to enhance the capacity and coverage of cellular networks.

Appendix A. Proof of Lemma 1

The probability of $\Delta_k > \delta$ given the UE is served by BS is [40]

$$P\left[\Delta_k > \delta | k = BS\right] = \frac{P\left[\Delta_k > \delta, k = BS\right]}{P[k = BS]} \tag{38}$$

where $P[k = BS] = A^{BS}$ is the corresponding association probability. The joint probability can be computed as

$$\begin{aligned} & P[\Delta_k > \delta, k = BS] \\ & = P[\Delta_k > \delta, P_j^{BS} C_j ||\delta^{BS}||^{-\alpha_j} > P_i^R C_i ||\delta^R||^{-\alpha_i}] \\ & = \int_\delta^\infty P[P_j^{BS} C_j ||\delta^{BS}||^{-\alpha_j} > P_i^R C_i ||\delta^R||^{-\alpha_i}] f_{\delta^{BS}}(r)\, dr \\ & \overset{(a)}{=} 2\pi\lambda_{BS} \int_\delta^\infty \exp\left(-\pi\left(\lambda_R \left(\frac{P_{i*}^R C_{i*}}{P_{j*}^{BS} C_{j*}}\right)^{2/\alpha_{i*}} r^{2\alpha_{j*}/\alpha_{i*}}\right.\right.\end{aligned}$$

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$$+\lambda_{BS}r^2\Big)\Big)r\,dr \qquad (39)$$

where (a) is computed similarly as (13). The CDF of Δ_k is $F_{\Delta_k}(\delta) = 1 - P[\Delta_k > \delta | k = BS]$ and the PDF is

$$f_{\Delta_k}(\delta) = \frac{dF_{\Delta_k}(\delta)}{d\delta} = \frac{2\pi\lambda_{BS}}{A^{BS}}\delta \exp\left(-\pi\left(\lambda_R\left(\frac{P^R_{i*}C_{i*}}{P^{BS}_{j*}C_{j*}}\right)^{2/\alpha_{i*}}\delta^{2\alpha_{j*}/\alpha_{i*}} + \lambda_{BS}\delta^2\right)\right) \qquad (40)$$

Similar analysis can be performed for a typical user associated with $\mathcal{R}$.

Appendix B. Proof of Lemma 2

The probability of $\Delta_k > \delta$ given the UE is served by BS is [40]

$$P[\Delta_k > \delta | k = BS] = \frac{P[\Delta_k > \delta, k = BS]}{P[k = BS]} \qquad (41)$$

where $P[k = BS] = A^{BS}_i$ is the corresponding association probability. The joint probability can be computed for multi-flow CA as

$$\begin{aligned} P[\Delta_k > \delta, k = BS] &= P[\Delta_k > \delta, P^{BS}_i C_i ||\delta^{BS}||^{-\alpha_i} > P^R_i C_i ||\delta^R||^{-\alpha_i}] \\ &= \int_\delta^\infty P[P^{BS}_i C_i ||\delta^{BS}||^{-\alpha_i} > P^R_i C_i ||\delta^R||^{-\alpha_i}] f_{\delta^{BS}}(r)\,dr \\ &\overset{(a)}{=} 2\pi\lambda_{BS}\int_\delta^\infty \exp\left(-\pi\left(\lambda_R\left(\frac{P^R_i}{P^{BS}_i}\right)^{2/\alpha_i} + \lambda_{BS}\right)r^2\right) r\,dr \end{aligned} \qquad (42)$$

where (a) is computed similarly as (28). The CDF of Δ_k is $F_{\Delta_k}(\delta) = 1 - P[\Delta_k > \delta | k = BS]$ and the PDF is

$$f_{\Delta_k}(\delta) = \frac{dF_{\Delta_k}(\delta)}{d\delta} = \frac{2\pi\lambda_{BS}}{A^{BS}_i}\delta \exp\left(-\pi\left(\lambda_R\left(\frac{P^R_i}{P^{BS}_i}\right)^{2/\alpha_i} + \lambda_{BS}\right)\delta^2\right) \qquad (43)$$

Similar analysis can be performed for a typical user associated with $\mathcal{R}$.

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